The Lossless Transmission Line

Say a transmission line is **lossless** (i.e., $R = G = 0$); the transmission line equations are then **significantly** simplified!

Characteristic Impedance

$$
\begin{aligned}
Z_0 &= \sqrt{\frac{R + j\omega L}{G + j\omega C}} \\
&= \sqrt{\frac{j\omega L}{j\omega C}} \\
&= \sqrt{\frac{L}{C}}
\end{aligned}
$$

Note the characteristic impedance of a **lossless** transmission line is purely **real** (i.e., Im{Z_0} =0)!

Propagation Constant

$$
\begin{aligned}
\gamma &= \sqrt{(R + j\omega L)(G + j\omega C)} \\
&= \sqrt{(j\omega L)(j\omega C)} \\
&= \sqrt{-\omega^2 LC} \\
&= j\,\omega\sqrt{LC}
\end{aligned}
$$

The wave propagation constant is purely **imaginary**!

In other words, for a **lossless** transmission line:

$$\alpha = 0 \quad \text{and} \quad \beta = \omega\sqrt{LC}$$

Note that since $\alpha = 0$, **neither** propagating wave is **attenuated** as they travel down the line—a wave at the **end** of the line is as large as it was at the **beginning**!

And this **makes sense**!

Wave attenuation occurs when **energy is extracted** from the propagating wave and turned into **heat**. This can **only** occur if resistance and/or conductance are present in the line. If $R = G = 0$, then **no attenuation** occurs—that why we call the line **lossless**.

Voltage and Current

The **complex functions** describing the magnitude and phase of the voltage/current at every location z along a transmission line are for a **lossless** line are:

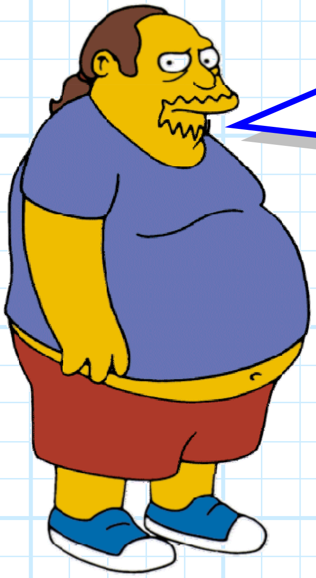
$$V(z) = V_0^+ \, e^{-j\beta z} + V_0^- \, e^{+j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}$$

Wavelength and Phase Velocity

We can now **explicitly** write the wavelength and propagation velocity of the two transmission line waves in terms of transmission line parameters L and C:

$$\lambda = \frac{2\pi}{\beta} = \frac{1}{f\sqrt{LC}} \qquad v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$



Q: *Oh **please**, continue wasting my valuable time. We both know that a **perfectly** lossless transmission line is a physical **impossibility**.*

A: True! However, a **low-loss** line **is** possible—in fact, it is **typical**! If $R \ll \omega L$ and $G \ll \omega C$, we find that the lossless transmission line equations are excellent **approximations**!

Unless **otherwise** indicated, **we will use the lossless equations** to **approximate** the behavior of a **low-loss** transmission line.

The lone **exception** is when determining the attenuation of a **long** transmission line. For that case we will use the approximation:

$$\alpha \approx \frac{1}{2}\left(\frac{R}{Z_0} + GZ_0\right)$$

where $Z_0 = \sqrt{L/C}$.

A summary of lossless transmission line equations

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \gamma = j\omega\sqrt{LC}$$

$$V(z) = V_0^+ \, e^{-j\beta z} + V_0^- \, e^{+j\beta z} \qquad I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{+j\beta z}$$

$$V^+(z) = V_0^+ \, e^{-j\beta z} \qquad V^-(z) = V_0^- \, e^{+j\beta z}$$

$$\beta = \omega\sqrt{LC} \qquad \lambda = \frac{1}{f\sqrt{LC}} \qquad v_p = \frac{1}{\sqrt{LC}}$$